

Solutions to Mathematics Workbook-1 | Quadratic Equations & Inequalities [2022]

Level - 2	Daily Tutorial Sheet 8
------------------	-------------------------------

96.(D) $x^2 + x + a = 0$

For roots to be greater than 'a' :

$$D \geq 0 \quad \Rightarrow \quad 1 - 4a \geq 0 \quad \Rightarrow \quad a \leq \frac{1}{4}$$

$$f(a) > 0 \quad \Rightarrow \quad a^2 + 2a > 0 \quad \Rightarrow \quad a(a+2) > 0 \Rightarrow a \in (-\infty, -2) \cup (0, \infty)$$

$$\text{Sum of roots} > 2a \quad \Rightarrow \quad a < -\frac{1}{2}$$

Combined solution:

$$a \in (-\infty, -2)$$

97.(D) $4x^2 - 20px + (25p^2 + 15p - 66) = 0$

For both the roots to be less than 2 : $D \geq 0$, $f(2) > 0$, $2 > \frac{-b}{2a}$

$$D \geq 0 \quad \Rightarrow \quad (20p)^2 - 4 \times 4 (25p^2 + 15p - 66) \geq 0$$

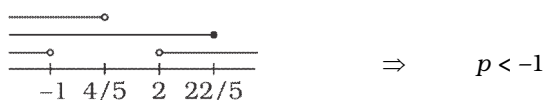
$$\Rightarrow 25p^2 - (25p^2 + 15p - 66) \geq 0 \Rightarrow -5p + 22 \geq 0 \Rightarrow p \leq \frac{22}{5} \quad \dots \text{(i)}$$

$$f(2) > 0 \quad \Rightarrow \quad 4(2)^2 - 20p(2) + 25p^2 + 15p - 66 > 0$$

$$\Rightarrow 25p^2 - 25p - 50 > 0 \Rightarrow p^2 - p - 2 > 0 \Rightarrow (p+1)(p-2) > 0 \quad \dots \text{(ii)}$$

$$2 > \frac{-b}{2a} \Rightarrow 2 > -\frac{(-20p)}{2 \times 4} \Rightarrow p < \frac{4}{5} \quad \dots \text{(iii)}$$

Combined solution :



98.(C) $x^2 - (a-3)x + a = 0$

$$D \geq 0$$

$$\Downarrow$$

$$f(2) > 0,$$

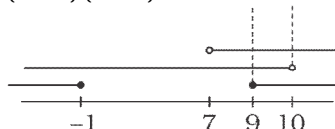
$$\Downarrow$$

$$2 < \frac{-b}{2a} \Rightarrow 2 < \frac{-(-(a-3))}{2}$$

$$(a-3)^2 - 4(a) \geq 0 \quad 4 - 2(a-3) + a > 0 \quad \Rightarrow a > 7$$

$$\Rightarrow a^2 - 10a + 9 \geq 0 \quad \Rightarrow a < 10$$

$$\Rightarrow (a-9)(a-1) \geq 0$$



Combined solution: $a \in [9, 10]$

99.(B) $x^2 - (m+1)x + (m+4) = 0$

$D \geq 0$

$\Rightarrow (m+1)^2 - 4(m+4) \geq 0 \Rightarrow m^2 - 2m - 15 \geq 0$

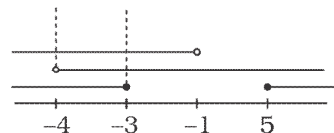
$\Rightarrow (m-5)(m+3) \geq 0 \dots \text{(i)}$

Sum of roots < 0

$\Rightarrow (m+1) < 0 \Rightarrow m < -1 \dots \text{(ii)}$

Product of roots $> 0 \Rightarrow m+4 > 0 \Rightarrow m > -4 \dots \text{(iii)}$

Combined Solution : $m \in (-4, -3]$



100.(B) sum of roots $> 2c \Rightarrow c < \frac{-a}{2} \Rightarrow$ (C) is false

$D \geq 0 \Rightarrow a^2 - 4b^2 \geq 0 \Rightarrow$ (A) is false

$f(c) > 0 \Rightarrow c^2 + ac + b^2 > 0 \Rightarrow$ (B) is true

101.(B) $2x^2 - (a^3 + 8a - 1)x + a^2 - 4a = 0$

Product of roots $= \frac{a^2 - 4a}{2} < 0 \Rightarrow a(a-4) < 0$ $a \in (0, 4)$

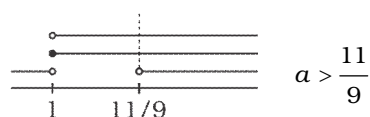
102.(C) $x^2 - 6ax + 2 - 2a + 9a^2 = 0$

$D \geq 0 \Rightarrow 36a^2 - 4(2 - 2a + 9a^2) \geq 0 \Rightarrow -4(2 - 2a) \geq 0 \Rightarrow a \geq 1 \dots \text{(i)}$

$f(3) > 0 \Rightarrow 9 - 18a + 2 - 2a + 9a^2 > 0 \Rightarrow 9a^2 - 20a + 11 > 0 \Rightarrow (9a - 11)(a - 1) > 0 \dots \text{(ii)}$

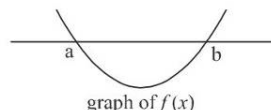
$3 < -\frac{(-6a)}{2} \Rightarrow a > 1 \dots \text{(iii)}$

Combined solution:

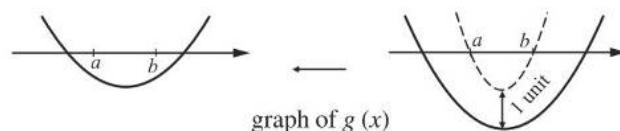


103.(D) Let $f(x) = (x-a)(x-b)$ and $g(x) = (x-a)(x-b) - 1$

Graph of $f(x)$ opens up as coefficient of x^2 is positive and it intersects x-axis at a and b.



Graph of $g(x)$ can be drawn by shifting graph of $f(x)$ one unit down as shown in the figure



Using graph of $g(x)$ we can observe that roots of $g(x) = 0$ lie outside a and b i.e. one root lies in $(-\infty, a)$ and other root lies in (b, ∞)

104.(B) For the first graph, $a > 0, c > 0$ as the parabola opens upwards and cuts the y-axis at positive side. From x-coordinate of vertex,

$$\text{We get } \frac{-b}{2a} > 0 \Rightarrow -b > 0 (a > 0) \Rightarrow b < 0$$

For the second graph $a > 0$ as the parabola opens upwards. $c < 0$ as the parabola cuts y-axis at negative side. x-coordinate of vertex $\Rightarrow \frac{-b}{2a} > 0 \Rightarrow -b > 0 (as a > 0) \Rightarrow b < 0$

For the third graph, $a > 0$ as the parabola opens upwards. $c > 0$ as the parabola cuts y-axis at negative side.

$$\text{x-coordinate of vertex} = \frac{-b}{2a} > 0 \Rightarrow b < 0 \quad (a > 0)$$

For the fourth graph,

$a < 0$: As parabola opens downwards

$c < 0$: As parabola cuts Y-axis at negative side

$$\text{X-coordinate of vertex} = -\frac{b}{2a} > 0 \Rightarrow b > 0 \quad (a < 0)$$

105.(C) Since α, β are roots of $(x-a)(x-b)-c=0$, therefore, by factor theorem.

$$(x-a)(x-b)-c = 1(x-\alpha)(x-\beta) \Rightarrow (x-\alpha)(x-\beta)+c = (x-a)(x-b)$$